



FIGURE 11. MAXIMUM PRESSURE-TO-STRENGTH RATIO,  $p/\sigma$ , IN MULTI-RING CONTAINER DESIGNED ON BASIS OF FATIGUE SHEAR STRENGTH

Each ring is assumed to be of the same ductile material.

In the formulation of the tensile fatigue criterion the parameter  $\alpha$  has been defined by Equation (13a). Thus, from Equations (13a) and (46) it is found that

$$\frac{p}{\sigma_1} = 2\alpha_r \frac{K^2 - 1}{K^2 + 1}, \quad \sigma_1 \leq \sigma_u \quad (47)$$

where  $\sigma_u$  is the ultimate tensile stress of the liner. The ratio  $p/\sigma_1$  is plotted in Figure 12 for various  $K$  and  $\alpha_r$ .

The fatigue data at room temperature of high-strength steels ( $\sigma_u \leq 300,000$  psi) listed previously in Tables 8, 9, and 10 are generally for  $\alpha_r \leq 0.5$  for lifetimes of  $10^4$  and greater. Hence, it is concluded that the maximum repeated pressure possible in a multi-ring container with a liner of  $\sigma_u = 300,000$  psi is approximately 300,000 psi if appreciable fatigue life is required. This conclusion presupposes that the outer components can also be designed to withstand the required interface pressure and that sufficient precompression can be provided in the liner so that  $\alpha_r = 0.5$  can be expected to give up to  $10^4$  cycles life. This is investigated next.

The stress range parameter  $\alpha_r$  depends on the mean stress parameter  $\alpha_m$ . The mean stress depends not only on the bore pressure  $p$  but on the interface pressures  $p_1$  and  $q_1$  between the liner and the second cylinder. The magnitudes of  $p_1$  and  $q_1$  that are possible depend upon the geometry and strength of the outer cylinders.

The outer rings are assumed to be all made of the same ductile material. Conducting a fatigue shear strength analysis of a multi-ring container having a pressure fluctuating between  $q_1$  and  $p_1$ , we find from a method similar to that used in arriving at Equation (42) (using Equation (40) for  $n = 2, 3, \dots, N-1$ ), that in this case also the optimum design has

$$k_2 = k_3 = \dots = k_n \quad (48)$$

Calculating the mean stress  $\sigma_m$  at the bore of the liner, equating  $\alpha_m \sigma_1$  to  $\sigma_m$  from Equation (13b), substituting for  $q_1$  from Equation (35), eliminating  $\sigma_1$  by use of Equation (47), and solving for  $p_1$ , one finds

$$p_1 = \frac{p}{K^2 - 1} \left[ \frac{K^2 - k_1^2}{k_1^2} + \frac{(K^2 + 1)}{4} \frac{(k_1^2 - 1)}{k_1^2} \frac{(\alpha_r - \alpha_m)}{\alpha_r} \right] \quad (49)$$

The other interface pressures  $p_n$ ,  $n \geq 2$  are again given by Equation (41). Eliminating the pressures  $p_1$  and  $p_n$ ,  $n \geq 2$  from Equations (49) and (41), and solving for the pressure-to-strength ratio  $p/\sigma$ , one gets

$$\frac{p}{\sigma} = \frac{2(K^2 - 1)(k_n^2 - 1)(N-1)k_1^2 \alpha_r}{k_n^2 \left[ 5(K^2 - k_1^2) + (\alpha_r - \alpha_m)(K^2 + 1)(k_1^2 - 1) \right]} \quad (50)$$